The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, SPRING SEMESTER 2019-2020

THERMODYNAMICS AND FLUID MECHANICS 2

Solutions

SECTION A
ANSWER ALL QUESTIONS IN THIS SECTION

1. Page 16 has the dry air tables, and for 300 K the value of conductivity, k, is 3.365×10^{-5} kW/m K or 0.03365 W/m K. The specific heat capacity at constant pressure is c_p which at 400 K for air is 1.0135 kJ/kg K. Density is 0.8824 kg/m^3. Dynamic viscosity, μ, is 2.268 kg/m s. 1 mark for each datum. [4]

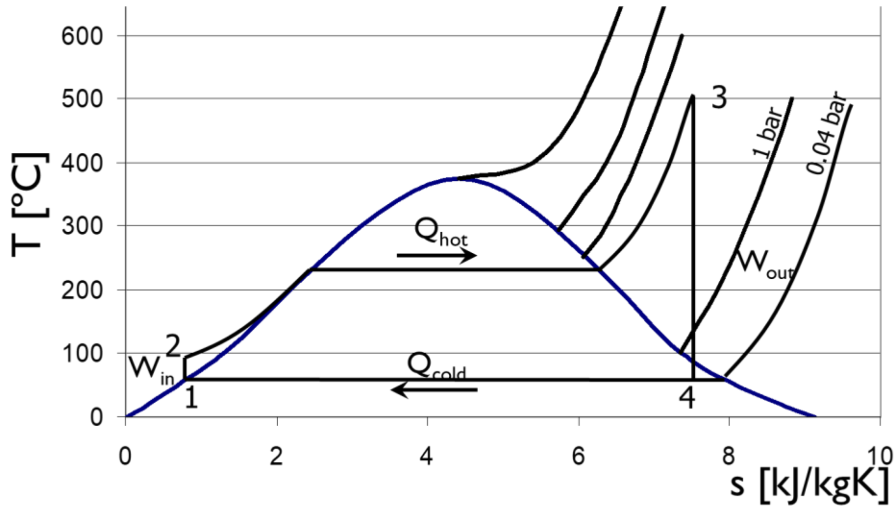
2. The condenser is at high pressure and high temperature, and the refrigerant at exit from this heat exchanger will be a liquid [1]. The throttle immediately after the condenser is a flow restrictor which reduces the pressure almost instantaneously [1]. The pressure drop in a refrigeration cycle is sufficient to lower the saturation pressure to a point where the refrigerant saturation temperature is appropriately cold [1]. The usual exit condition has a significant mass fraction of vapour and is immediately at the cold temperature of the boiling point of the saturated vapour at that point [1].

3.

	ni/n	M_i (kg/kmol)	ni/n*M_i	$m_i/m=n_i/n*M_i/M$	c_{pi} (kJ/kjk)	c_{pi}*mi/m
CH_4	0.7	16	11.2	0.459	2.226	1.02
CO_2	0.3	44	13.2	0.541	0.846	0.457
	1.0	M=	24.4 kg/kmol [1]	1.000	c_p=	1.304 kJ/kgK [1]

Gas constant $R=R_u/M$ = 8.3145 kJ/kmol k /24.4 kg/kmol = 0.341 kJ/kgK [1]
Gas constant R= Cp-Cv, thus Cv = Cp-R = 1.304-0.341 = 0.962 kJ/kgK [1]

4.

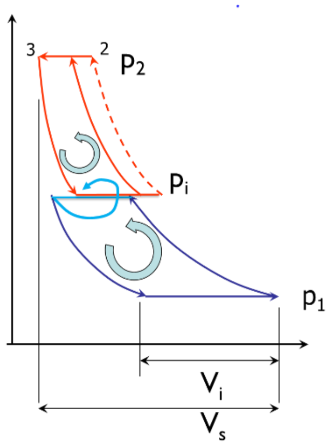


1 mark for each process correctly identified, 1-2, 2-3, 3-4, 4-1 according to question.

5. Prandtl and Grashof numbers are significant for convection driven flows, and Reynolds number is not
[1]
Prandtl number relates viscous diffusion to thermal diffusion and Grashof number relates the buoyancy to viscous forces, so both are related to heat induced fluids flow

[2]
Reynolds number relates momentum induced flow to the viscous forces and is for forced flows
[1]

6. Cooling the gas reduces the work done [1], and minimum work is achieved with isothermal compression [1]. Therefore cooling the air between the two stages of compression in an intercooler heat exchanger will improve the work efficiency of the process [1]. Work saved is indicated in the diagram by the dotted line showing the adiabatic process on 2nd stage here which gives a larger area in the integral than the intercooled compression solid line. [1]



7. (a) First calculate the Reynolds number:

$$Re_L = \frac{\rho U L}{\mu} = \frac{1.2\ \frac{kg}{m^3} \cdot 50\ \frac{m}{s} \cdot 2\ m}{1.8 \cdot 10^{-5}\ \frac{kg}{m \cdot s}} = 6.67 \cdot 10^6$$

Using now Prandtl's formula for the drag coefficient:

$$C_D = \frac{0.031}{Re_L^{1/7}} = 0.00328$$

And the drag force:

$$D = \tfrac{1}{2} C_D \rho U_0^2 b L = \tfrac{1}{2} \cdot 0.00328 \cdot 1.2\ \tfrac{kg}{m^3} \cdot \left(50\ \tfrac{m}{s}\right)^2 \cdot 0.5\ m \cdot 2\ m = 4.92\ N$$

[2]

(b) Using Schlichting equation:

$$C_D = \frac{0.031}{Re_L^{1/7}} - \frac{8700}{Re_L} = \frac{0.031}{(6.67\cdot 10^6)^{\frac{1}{7}}} - \frac{8700}{6.67\cdot 10^6} = 0.00198$$

And the drag force:

$$D = \frac{1}{2}C_D \rho U_0^2 bL = \frac{1}{2}\cdot 0.00198\cdot 1.2\,\frac{kg}{m^3}\cdot\left(50\,\frac{m}{s}\right)^2\cdot 0.5\,m\cdot 2\,m = 2.97\,N$$

[2]

8. There are 5 variables 3 reference dimensions (M, L, T). The number of pi groups will be 5 – 3 = 2. [4]

9. The angle of Mach cone can be estimated from sin α = c / V where α is the angle.
sin α = 340/680 = 0.5 [2]

it follows that α is equal to 30°. [2]

10.

$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right]$ is the <u>inertial force</u> along y per unit volume of fluid [1.5]

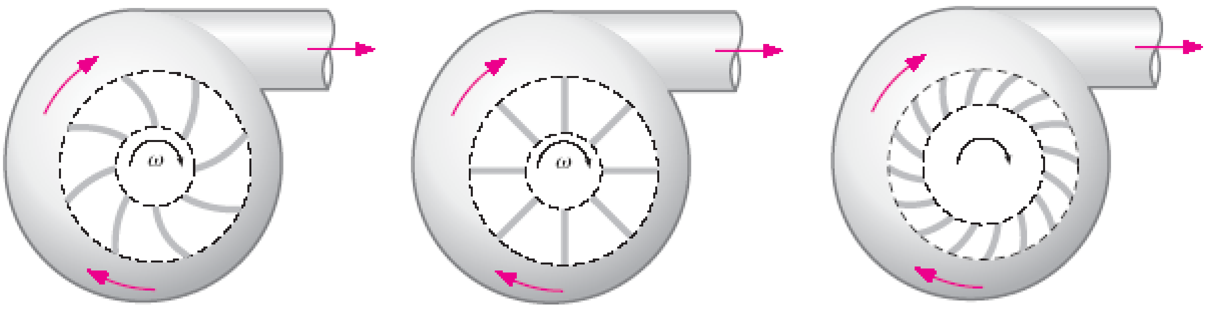
$\frac{\partial p}{\partial y}$ is the <u>pressure force</u> along y per unit volume of fluid [1]

$\mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$ is the <u>viscous force</u> along y per unit volume of fluid [1.5]

11. Because the aircraft if cruising at constant speed it means that the thrust force has to be the same as the drag force. Drag force can be expressed in the following way:

D = ½ ρ V^2 Area C_D
Hence: Thrust = ½ ρ V^2 Area C_D
The drag coefficient is: C_D = 2 T / (ρ V^2 Area) [2]
C_D = 2 x 200000 / (1 x 200^2 x 100)
C_D = 0.1 [2]

12. The three main type of centrifugal pumps are:



a-Backward inclined blades [1] b-Radial/straight blades [1] c-Forward-inclined blades [1]
And 1 mark for decent images with rotation direction.

SECTION B
ANSWER 2 QUESTIONS FROM THIS SECTION

13.

a) The two forces are drag [1 mark] and gravity [1 mark]. [2]

b) Drag force:

$$D = \frac{1}{2} C_D \rho U^2 L^2,$$

gravitational force:

$$G = mg.$$

Therefore, the free-fall velocity is obtained by the condition that $D = G$:

$$U = \sqrt{\frac{2mg}{C_D \rho L^2}} = \sqrt{\frac{2 \cdot 20\ kg \cdot 9.81\ m/s^2}{1.07 \cdot 1.29\ kg/m^3 \cdot (0.3\ m)^2}} = 56.2 \frac{m}{s}.$$

Now it should be checked that $Re \geq 10^4$, to allow the use of the data in Table T13:

$$Re = \frac{\rho U L}{\mu} = \frac{1.29\ kg/m^3 \cdot 56.2\ m/s \cdot 0.3\ m}{1.71 \cdot 10^{-5}\ kg/(m\ s)} = 1.27 \cdot 10^6 \geq 10^4\ OK.$$

Marking scheme: [2 mark] for the correct formula for the drag force D, in particular for the right expression of the frontal area; [1 mark] for extracting the correct value of $C_D = 1.07$ from the table; [1 mark] for extracting the velocity U by equating $D = G$, and [1 mark] for getting the correct numerical value; [1 mark] for the final check on the Reynolds number. [6]

c) With the parachute, the force balance (weight of the parachute neglected) becomes:

$$\frac{1}{2} C_{D,box} \rho U^2 L^2 + \frac{1}{2} C_{D,par} \rho U^2 \frac{\pi d^2}{4} = mg,$$

and the diameter of the parachute necessary to limit the fall velocity to $U = 10\ m/s$ *is:*

$$d = \sqrt{\frac{8\left(mg - \frac{1}{2} C_{D,box} \rho U^2 L^2\right)}{C_{D,par} \rho U^2 \pi}}$$

$$= \sqrt{\frac{8(20\ kg \cdot 9.81\ m/s^2 - 0.5 \cdot 1.07 \cdot 1.29\ kg/m^3 \cdot (10\ m/s)^2 \cdot (0.3\ m)^2)}{1.2 \cdot 1.29\ kg/m^3 \cdot (10\ m/s)^2 \pi}}$$

$$= 1.79\ m.$$

Marking scheme: [2 mark] for the correct frontal area of the parachute; [1 mark] for the correct value of $C_D = 1.2$ for the parachute from the table; [2 mark] for the correct formula or numerical value of the parachute diameter. [5]

14.

The general functional relationship is:

F = *f* (h, w, ρ, μ, V)

Which has 6 variables and 3 reference dimensions (MLT)

F = M L T^{-2}
h = L
w = L
ρ = M L^{-3}
μ = M L^{-1} T^{-1}
V = L T^{-1}

We will get three pi groups.

We select three repeating variables: h, V, and ρ.

The first pi group is:

$$\Pi_1 = F\, h^a\, V^b\, \rho^c$$

$$M^0L^0T^0 = (MLT^{-2})(L)^a(LT^{-1})^b(ML^{-3})^c$$

(b)

M) 1 + c = 0 c = -1
L) 1 + a + b -3c = 0 a = -1 –b +3c = -1 +2 -3 = -2
T) -2 –b = 0 b = -2

[3]

It follows:

$$\Pi_1 = F\, h^{-2}\, V^{-2}\, \rho^{-1} = \frac{F}{h^2\, V^2\, \rho}$$

[4 marks]

For the second pi group by inspection of the variable we can say that the ratio h over w will be another pi group.

$$\Pi_2 = \frac{h}{w}$$

[3 marks]

For the third group we can write:

$$\Pi_3 = \mu\, h^a\, V^b\, \rho^c$$

$$M^0L^0T^0 = (M\, L^{-1}\, T^{-1})(L)^a(LT^{-1})^b(ML^{-3})^c$$

M) 1 + c = 0 c = -1
L) -1 + a + b -3c = 0 a = 1 –b +3c = 1 +1 -3 = -1
T) -1 –b = 0 b = -1

It follows:

$$\Pi_3 = \mu\, h^{-1}\, V^{-1}\, \rho^{-1} = \frac{\mu}{h\, V\, \rho}$$

[3 marks]

15.

i) To evaluate the maximum mass flow rate we use the capacity coefficient:

$$C_Q = \frac{Q}{nD^3}$$

$$Q = C_Q n\, D^3 = 0.1\ x\ 1000\ x\ (0.5)^3 = 12.5\ m^3/s$$

$$\dot{m}_{max} = Q\ x\ \rho_0 = 12.5\ x\ 1.394\ =\ 17.4\ \mathrm{kg/s}$$

ii) For the flow to be supersonic at exit the flow at the throat must be sonic. The maximum mass flow rate at the throat can be evaluated by the following:

$$\dot{m}_{max} = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \rho_0 A^* \sqrt{RT_0}$$

And

$$\rho_0 = \frac{P_0}{R\, T_0} = 1.394$$

The area of the throat is:

$$A^* = \frac{\dot{m}_{max}}{\sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \rho_0 \sqrt{RT_0}}$$

$$A^* = \frac{17.4}{\sqrt{1.4}\left(\frac{2}{2.4}\right)^{\frac{2.4}{2(0.4)}} 1.394\sqrt{287\ x\ 500}} = 0.048\ m^2$$

$$d = \sqrt{\frac{4A^*}{\pi}} = 0.248\ m$$

iii) The flow is isentropic and so P_0 and T_0 do not vary through the flow. We get the exit pressure from

$$\frac{p_o}{p_e} = \left\{1 + Ma^2\left(\frac{\gamma-1}{2}\right)\right\}^{\frac{\gamma}{\gamma-1}} = \{1 + 2.5^2 \times 0.2\}^{\frac{1.4}{0.4}}$$
$$= 17.09$$

$$p_e = \frac{200}{17.09} = 11.7kPa$$

iv) Exit temperature obtained from

$$\frac{T_o}{T_e} = 1 + Ma^2\left(\frac{\gamma-1}{2}\right) \qquad T_e = \frac{500}{1 + 2.5^2(0.2)} = 222K$$

SECTION C

ANSWER 2 QUESTIONS FROM THIS SECTION

16.(a)

i) The stoichiometric equation with N_2 is:

$$0.6C_3H_8 + 0.4C_4H_4 + 5O_2 + 18.8N_2 \rightarrow 3.4CO_2 + 3.2H_2O + 18.8N_2$$

With 15% excess air, total air is 115% or 1.15 times the stoichiometric air, then the equation will be:

$$0.6C_3H_8 + 0.4C_4H_4 + 1.15 \times 5O_2 + 1.15 \times 18.8N_2$$
$$\rightarrow 3.4CO_2 + 3.2H_2O + 1.15 \times 18.8N_2 + 0.15 \times 5.0\ O_2$$

And completing the products:

$$0.6C_3H_8 + 0.4C_4H_4 + 5.75O_2 + 21.6N_2 \rightarrow 3.4CO_2 + 3.2H_2O + 21.6N_2 + 0.75O_2$$

[3]

Air to fuel ratio by volume is: (5.75+21.6)/1 = 27.4 [2]

ii)

	ni (kmol)	ni/n (wet)		ni (dry)	ni/n (dry)
CO_2	3.4	0.117		3.4	0.132
H_2O	3.2	0.110		-	-
O_2	0.75	0.026		0.75	0.029
N_2	21.6	0.745		21.6	0.837
n(wet)=	29.0	1.000	n(dry)=	25.8	1.000

[2]

(b)

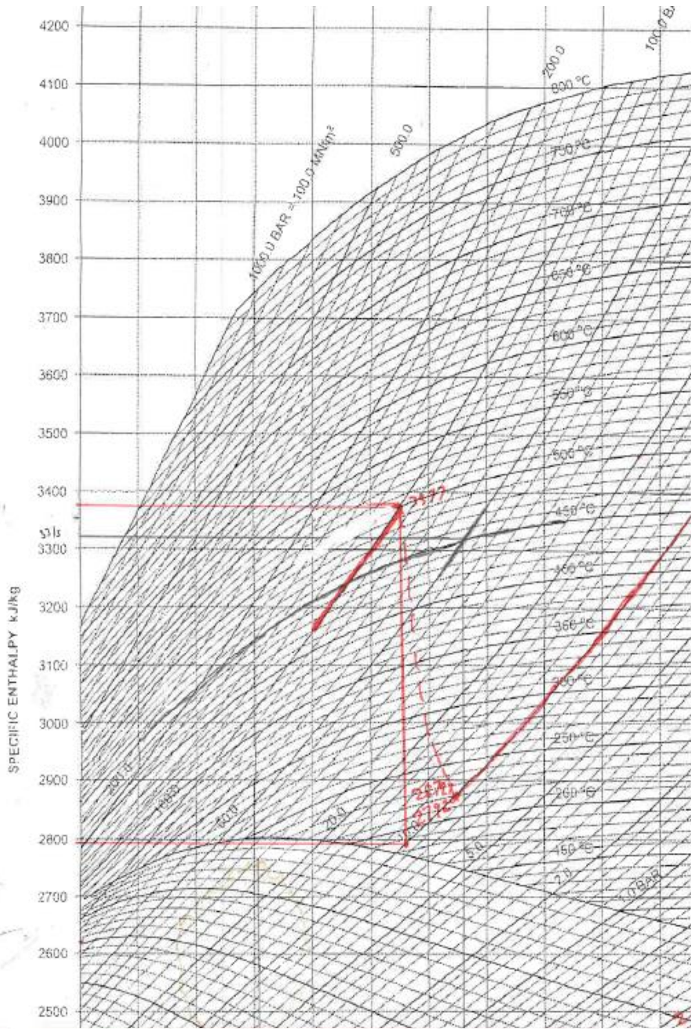
i) At the inlet of HP Turbine P_1= 100 bar and temperature is 500°C, from the chart h1 = 3377 kj/kgk. [1]

At the exit of the HP turbine P_2= 10 bar, using isentropic expansion, vertical line on the chart, $h_{2'}$ = 2792 kJ/kgK. [1]

Isentropic efficiency of the HP turbine is 0.85, then specific enthalpy at the exit of the turbine is,

h2 =3377-(3377-2792)*0.85 = 2880 kJ/kgK, [1]

Chart used to confirm that the method is correct:



ii) power out is:

$$\dot{W} = \dot{m}\Delta h$$

$$\dot{W} = 50 \times (3377 - 2880) = 24850\,\text{kW}$$

[2]

Therefore thermal efficiency:

$$\eta_{Th} = \frac{\dot{W}}{\dot{Q}} = \frac{24850}{160000} = 0.155$$

[1]

17.(a)

i) from the formula sheet:

$$p_i = \sqrt{p_1 p_2}$$

Therefore intermediate pressure is:

$$p_i = \sqrt{1 \times 16} = 4\ bar$$

[2]

ii) steady polytropic work equation is:

$$\dot{W} = \dot{m}R\frac{n}{n-1}[T_2 - T_1]$$

to calculate the work it is first necessary to calculate T_2 using the formula for polytropic compression which is adjusted from the adiabatic compression formula:

$$\frac{T_2}{T_1} = \left[\frac{p_2}{p_1}\right]^{\frac{n-1}{n}}$$

$$\frac{T_2}{290} = [4]^{\frac{1.3-1}{1.3}} \rightarrow T_2 = 290 \times [4]^{0.231} = 399\ \mathrm{K}$$

[2]

And the work is, using the gas constant of approximate air:

$$\dot{W} = 0.05 \times 0.287\frac{1.3}{1.3-1}[399 - 290] = 6.78\ \mathrm{kW}$$

[2]

(b)

i) At 70% RH,

$$\phi = \frac{p_s}{p_{sat}} = 0.7$$

p_{sat} is the saturation pressure of water vapour at the temperature of the air, 25℃; p_g = 0.03166 bar.

Therefore p_s = 0.7*0.03166 = 0.02216 bar

Dew point is where p_s = p_{sat}, and the temperature will be 15℃ [4]

ii) Specific humidity,

$$\omega = \frac{0.622 p_s}{p_a - p_s} = \frac{m_s}{m_a}$$

$$\omega = \frac{0.622 \times 0.02216}{1 - 0.02216} = \frac{m_s}{0.15 - m_s}$$

Therefore ω = 0.0141 and m_s = 0.0021 kg/s [3]

18.(a)

i) Calculate Gr, initially putting in the known data:

$$Gr = \frac{g\beta L^3 \rho^2 \Delta T}{\mu^2};\ Gr = \frac{9.81 \times \beta \times 0.5^3 \rho^2 (45 - 9)}{\mu^2}$$

find the properties data in the data tables for air, identifying first that properties are worked out at the film temperature, i.e. the average of surface and surrounding air, 27°C or (conveniently) 300 K, on p.16:
density, ρ = 1.177 kg/m^3
dynamic viscosity, μ = 1.846×10^{-5} kg/ms
compressibility, $\beta = \frac{1}{T_f} = \frac{1}{300} = 0.003\dot{3}$ /K
and Prandtl number, Pr = 0.707

[2]

Therefore Gr:

$$Gr = \frac{9.81 \times 0.003\dot{3} \times 0.5^3 \times 1.177^2 (45 - 9)}{(1.846 \times 10^{-5})^2} = 5.982 \times 10^8$$

and Gr×Pr = 4.229×10^8
Therefore the correlation is ok to use. [2]

ii) The heat transfer rate is by the heat transfer coefficient which is calculated from the Nusselt number:

$$\mathrm{Nu} = 0.59(\mathrm{GrPr})^{0.25} = \frac{hd}{k}$$

$$\mathrm{Nu} = 0.59(4.229 \times 10^8)^{0.25} = 84.6 = \frac{h \times 0.5}{0.025}$$

Therefore h = 4.23 W/m^{2}K. [2]

(b)

i) Water flow capacity rate C_w = 0.15*4.18 = 0.627 kW/ °C
Oil flow capacity rate C_0 = 0.25*2.20 = 0.550 kW/ °C
Thus the minimum capacity rate is the oil capacity rate C_o = C_{min} = 0.550 kW/°C [2]
NTU for the heat exchanger is = UA/C_{min}

Where A is total the heat transfer area, A = 2.12 m^2
So, NTU = 0.350*2.12/0.550 = 1.35 [2]

ii) The maximum heat transfer Qmax = C_{min} * ΔT_{max} = 0.550 (150-15) = 74.25 kW
The actual heat transfer through the heat exchanger is Q= ε Q_{max}
Where ε is the effectiveness of the heat exchanger ε = 0.58,
The actual heat transfer Q = 0.58*74.25 = 43.1 kW [3]

END